Національний технічний університет України

«Київський політехнічний інститут імені Ігоря Сікорського»

Факультет інформатики та обчислювальної техніки

Кафедра обчислювальної техніки

Методи оптимізації та планування експерименту

Лабораторна робота №5

“ ПРОВЕДЕННЯ ТРЬОХФАКТОРНОГО ЕКСПЕРИМЕНТУ ПРИ ВИКОРИСТАННІ РІВНЯННЯ РЕГРЕСІЇ З УРАХУВАННЯМ КВАДРАТИЧНИХ ЧЛЕНІВ

(ЦЕНТРАЛЬНИЙ ОРТОГОНАЛЬНИЙ КОМПОЗИЦІЙНИЙ ПЛАН)”

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Київ

2020 р.

**Мета:** Провести трьохфакторний експеримент з урахуванням квадратичних членів, використовуючи центральний ортогональний композиційний план. Знайти рівняння регресії, яке буде адекватним для опису об'єкту.

Номер у списку: 10.

Варіант завдання: 310.



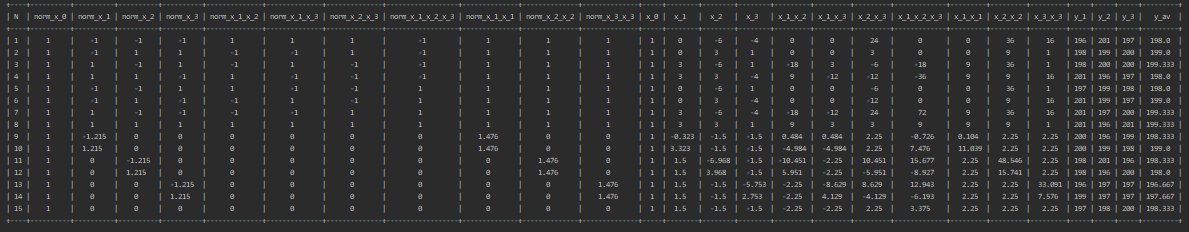


1. Лістинг програми:

from copy import deepcopy  
from math import sqrt  
import numpy as np  
from prettytable import PrettyTable  
  
x1\_min = 0  
x1\_max = 3  
x2\_min = -6  
x2\_max = 3  
x3\_min = -4  
x3\_max = 1  
  
x\_average\_max = (x1\_max + x2\_max + x3\_max) / 3  
x\_average\_min = (x1\_min + x2\_min + x3\_min) / 3  
y\_max = 200 + x\_average\_max  
y\_min = 200 + x\_average\_min  
  
  
def replace\_column(list\_: list, column, list\_replace):  
 list\_ = deepcopy(list\_)  
 for i in range(len(list\_)):  
 list\_[i][column] = list\_replace[i]  
 return list\_  
  
  
def append\_to\_list\_x(x: list, variant: int):  
 if variant == 1:  
 for i in range(len(x)):  
 x[i].append(x[i][1] \* x[i][2])  
 x[i].append(x[i][1] \* x[i][3])  
 x[i].append(x[i][2] \* x[i][3])  
 x[i].append(x[i][1] \* x[i][2] \* x[i][3])  
 if variant == 2:  
 for i in range(len(x)):  
 x[i].append(x[i][1] \* x[i][2])  
 x[i].append(x[i][1] \* x[i][3])  
 x[i].append(x[i][2] \* x[i][3])  
 x[i].append(x[i][1] \* x[i][2] \* x[i][3])  
 x[i].append(x[i][1] \* x[i][1])  
 x[i].append(x[i][2] \* x[i][2])  
 x[i].append(x[i][3] \* x[i][3])  
 for i in range(len(x)):  
 for j in range(len(x[i])):  
 if round(x[i][j], 3) == 0:  
 x[i][j] = 0  
 x[i][j] = round(x[i][j], 3)  
  
  
def get\_value(table: dict, key: int):  
 value = table.get(key)  
 if value is not None:  
 return value  
 for i in table:  
 if type(i) == range and key in i:  
 return table.get(i)  
  
  
def main(m, n):  
 if n == 15:  
 const\_l = 1.215  
 print(  
 'ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3 + b12 \* x1 \* x2 + b13 \* x1 \* x3 + b23 \* x2 \* x3 + b123 \* x1 \* x2 \* '  
 'x3 + b11 \* x1 \* x1 + b22 \* x2 \* x2 + b33 \* x3 \* x3')  
 norm\_x = [  
 [+1, -1, -1, -1],  
 [+1, -1, +1, +1],  
 [+1, +1, -1, +1],  
 [+1, +1, +1, -1],  
 [+1, -1, -1, +1],  
 [+1, -1, +1, -1],  
 [+1, +1, -1, -1],  
 [+1, +1, +1, +1],  
 [+1, -const\_l, 0, 0],  
 [+1, const\_l, 0, 0],  
 [+1, 0, -const\_l, 0],  
 [+1, 0, const\_l, 0],  
 [+1, 0, 0, -const\_l],  
 [+1, 0, 0, const\_l],  
 [+1, 0, 0, 0]  
 ]  
  
 delta\_x1 = (x1\_max - x1\_min) / 2  
 delta\_x2 = (x2\_max - x2\_min) / 2  
 delta\_x3 = (x2\_max - x3\_min) / 2  
 x01 = (x1\_min + x1\_max) / 2  
 x02 = (x2\_min + x2\_max) / 2  
 x03 = (x3\_min + x3\_max) / 2  
  
 x = [  
 [1, x1\_min, x2\_min, x3\_min],  
 [1, x1\_min, x2\_max, x3\_max],  
 [1, x1\_max, x2\_min, x3\_max],  
 [1, x1\_max, x2\_max, x3\_min],  
 [1, x1\_min, x2\_min, x3\_max],  
 [1, x1\_min, x2\_max, x3\_min],  
 [1, x1\_max, x2\_min, x3\_min],  
 [1, x1\_max, x2\_max, x3\_max],  
 [1, -const\_l \* delta\_x1 + x01, x02, x03],  
 [1, const\_l \* delta\_x1 + x01, x02, x03],  
 [1, x01, -const\_l \* delta\_x2 + x02, x03],  
 [1, x01, const\_l \* delta\_x2 + x02, x03],  
 [1, x01, x02, -const\_l \* delta\_x3 + x03],  
 [1, x01, x02, const\_l \* delta\_x3 + x03],  
 [1, x01, x02, x03]  
 ]  
  
 append\_to\_list\_x(norm\_x, variant=2)  
 append\_to\_list\_x(x, variant=2)  
  
 if n == 8:  
 print(  
 'ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3 + b12 \* x1 \* x2 + b13 \* x1 \* x3 + b23 \* x2 \* x3 + b123 \* x1 \* x2 \* x3'  
 )  
 norm\_x = [  
 [+1, -1, -1, -1],  
 [+1, -1, +1, +1],  
 [+1, +1, -1, +1],  
 [+1, +1, +1, -1],  
 [+1, -1, -1, +1],  
 [+1, -1, +1, -1],  
 [+1, +1, -1, -1],  
 [+1, +1, +1, +1]  
 ]  
  
 x = [  
 [1, x1\_min, x2\_min, x3\_min],  
 [1, x1\_min, x2\_max, x3\_max],  
 [1, x1\_max, x2\_min, x3\_max],  
 [1, x1\_max, x2\_max, x3\_min],  
 [1, x1\_min, x2\_min, x3\_max],  
 [1, x1\_min, x2\_max, x3\_min],  
 [1, x1\_max, x2\_min, x3\_min],  
 [1, x1\_max, x2\_max, x3\_max]  
 ]  
  
 append\_to\_list\_x(norm\_x, variant=1)  
 append\_to\_list\_x(x, variant=1)  
  
 if n == 4:  
 print('ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3')  
 norm\_x = [  
 [+1, -1, -1, -1],  
 [+1, -1, +1, +1],  
 [+1, +1, -1, +1],  
 [+1, +1, +1, -1],  
 ]  
 x = [  
 [1, x1\_min, x2\_min, x3\_min],  
 [1, x1\_min, x2\_max, x3\_max],  
 [1, x1\_max, x2\_min, x3\_max],  
 [1, x1\_max, x2\_max, x3\_min],  
 ]  
 y = np.random.randint(y\_min, y\_max, size=(n, m))  
 y\_av = list(np.average(y, axis=1))  
  
 for i in range(len(y\_av)):  
 y\_av[i] = round(y\_av[i], 3)  
  
 if n == 15:  
 t = PrettyTable(['N', 'norm\_x\_0', 'norm\_x\_1', 'norm\_x\_2', 'norm\_x\_3', 'norm\_x\_1\_x\_2', 'norm\_x\_1\_x\_3',  
 'norm\_x\_2\_x\_3', 'norm\_x\_1\_x\_2\_x\_3', 'norm\_x\_1\_x\_1', 'norm\_x\_2\_x\_2', 'norm\_x\_3\_x\_3', 'x\_0',  
 'x\_1', 'x\_2', 'x\_3', 'x\_1\_x\_2', 'x\_1\_x\_3', 'x\_2\_x\_3', 'x\_1\_x\_2\_x\_3', 'x\_1\_x\_1', 'x\_2\_x\_2',  
 'x\_3\_x\_3'] + [f'y\_{i + 1}' for i in range(m)] + ['y\_av'])  
  
 if n == 8:  
 t = PrettyTable(['N', 'norm\_x\_0', 'norm\_x\_1', 'norm\_x\_2', 'norm\_x\_3', 'norm\_x\_1\_x\_2', 'norm\_x\_1\_x\_3',  
 'norm\_x\_2\_x\_3', 'norm\_x\_1\_x\_2\_x\_3', 'x\_0', 'x\_1', 'x\_2', 'x\_3', 'x\_1\_x\_2', 'x\_1\_x\_3',  
 'x\_2\_x\_3', 'x\_1\_x\_2\_x\_3'] + [f'y\_{i + 1}' for i in range(m)] + ['y\_av'])  
 if n == 4:  
 t = PrettyTable(  
 ['N', 'norm\_x\_0', 'norm\_x\_1', 'norm\_x\_2', 'norm\_x\_3', 'x\_0', 'x\_1', 'x\_2', 'x\_3'] +  
 [f'y\_{i + 1}' for i in range(m)] + ['y\_av'])  
  
 for i in range(n):  
 t.add\_row([i + 1] + list(norm\_x[i]) + list(x[i]) + list(y[i]) + [y\_av[i]])  
 print(t)  
  
 m\_ij = []  
 for i in range(len(x[0])):  
 m\_ij.append([round(sum([x[k][i] \* x[k][j] for k in range(len(x))]) / 15, 3) for j in range(len(x[i]))])  
  
 k\_i = []  
 for i in range(len(x[0])):  
 a = sum(y\_av[j] \* x[j][i] for j in range(len(x))) / 15  
 k\_i.append(a)  
  
 det = np.linalg.det(m\_ij)  
 det\_i = [np.linalg.det(replace\_column(m\_ij, i, k\_i)) for i in range(len(k\_i))]  
  
 b\_i = [round(i / det, 3) for i in det\_i]  
 if n == 15:  
 print(  
 f"\nThe naturalized regression equation: "  
 f"y = {b\_i[0]:.5f} + {b\_i[1]:.5f} \* x1 + {b\_i[2]:.5f} \* x2 + "  
 f"{b\_i[3]:.5f} \* x3 + {b\_i[4]:.5f} \* x1 \* x2 + "  
 f"{b\_i[5]:.5f} \* x1 \* x3 + {b\_i[6]:.5f} \* x2 \* x3 + {b\_i[7]:.5f} \* x1 \* x2 \* x3 + {b\_i[8]:.5f} \* x1 \* x1 + "  
 f"{b\_i[9]:.5f} \* x2 \* x2 + {b\_i[10]:.5f} \* x3 \* x3")  
 if n == 8:  
 print(  
 f"\nThe naturalized regression equation: "  
 f"y = {b\_i[0]:.5f} + {b\_i[1]:.5f} \* x1 + {b\_i[2]:.5f} \* x2 + "  
 f"{b\_i[3]:.5f} \* x3 + {b\_i[4]:.5f} \* x1 \* x2 + "  
 f"{b\_i[5]:.5f} \* x1 \* x3 + {b\_i[6]:.5f} \* x2 \* x3 + {b\_i[7]:.5f} \* x1 \* x2 \* x3")  
 if n == 4:  
 print(  
 f"\nThe naturalized regression equation: "  
 f"y = {b\_i[0]:.5f} + {b\_i[1]:.5f} \* x1 + {b\_i[2]:.5f} \* x2 + {b\_i[3]:.5f} \* x3\n")  
  
 check\_i = [round(sum(b\_i[j] \* i[j] for j in range(len(b\_i))), 3) for i in x]  
 for i in range(len(check\_i)):  
 print(f'ŷ{i + 1} = {check\_i[i]}, y\_av{i + 1} = {y\_av[i]}')  
  
 print("\n[ Kohren's test ]")  
 f\_1 = m - 1  
 f\_2 = n  
 s\_i = [sum([(i - y\_av[j]) \*\* 2 for i in y[j]]) / m for j in range(len(y))]  
 g\_p = max(s\_i) / sum(s\_i)  
  
 table = {3: 0.6841, 4: 0.6287, 5: 0.5892, 6: 0.5598, 7: 0.5365, 8: 0.5175, 9: 0.5017, 10: 0.4884,  
 range(11, 17): 0.4366, range(17, 37): 0.3720, range(37, 145): 0.3093}  
 g\_t = get\_value(table, m)  
  
 if g\_p < g\_t:  
 print(f"The variance is homogeneous: Gp = {g\_p:.5} < Gt = {g\_t}")  
 else:  
 print(f"The variance is not homogeneous Gp = {g\_p:.5} < Gt = {g\_t}\nStart again with m = m + 1 = {m + 1}")  
 return main(m=m + 1, n=n)  
  
 print("\n[ Student's test ]")  
 s2\_b = sum(s\_i) / n  
 s2\_beta\_s = s2\_b / (n \* m)  
 s\_beta\_s = sqrt(s2\_beta\_s)  
 if n == 15:  
 beta\_i = b\_i  
 else:  
 beta\_i = [sum([norm\_x[i][j] \* y\_av[i] for i in range(len(norm\_x))]) / n for j in range(len(norm\_x[0]))]  
 beta\_i = [round(i, 3) for i in beta\_i]  
  
 t = [abs(i) / s\_beta\_s for i in beta\_i]  
  
 f\_3 = f\_1 \* f\_2  
 t\_table = {8: 2.306, 9: 2.262, 10: 2.228, 11: 2.201, 12: 2.179, 13: 2.160, 14: 2.145, 15: 2.131, 16: 2.120,  
 17: 2.110, 18: 2.101, 19: 2.093, 20: 2.086, 21: 2.08, 22: 2.074, 23: 2.069, 24: 2.064,  
 range(25, 30): 2.06, range(30, 40): 2.042, range(40, 60): 2.021, range(60, 100): 2,  
 range(100, 2 \*\* 100): 1.96}  
 d = deepcopy(n)  
 for i in range(len(t)):  
 if get\_value(t\_table, f\_3) > t[i]:  
 beta\_i[i] = 0  
 d -= 1  
 if n == d:  
 n = 8 if n == 4 else 15  
 print(f"n=d\nStart again with n = {n}")  
 return main(m=m + 1, n=n)  
 if n == 15:  
 print(  
 f"\nThe naturalized simplified regression equation: "  
 f"y = {beta\_i[0]:.5f} + {beta\_i[1]:.5f} \* x1 + "  
 f"{beta\_i[2]:.5f} \* x2 + {beta\_i[3]:.5f} \* x3 + {beta\_i[4]:.5f} \* x1 \* x2 + "  
 f"{beta\_i[5]:.5f} \* x1 \* x3 + {beta\_i[6]:.5f} \* x2 \* x3 + {beta\_i[7]:.5f} \* x1 \* x2 \* x3 + "  
 f"{beta\_i[8]:.5f} \* x1 \* x1 + {beta\_i[9]:.5f} \* x2 \* x2 + {beta\_i[10]:.5f} \* x3 \* x3")  
 check\_i = [round(sum(beta\_i[j] \* i[j] for j in range(len(beta\_i))), 3) for i in x]  
  
 if n == 8:  
 print(  
 f"\nThe normalized regression equation: "  
 f"y = {beta\_i[0]:.5f} + {beta\_i[1]:.5f} \* x1 + {beta\_i[2]:.5f} \* x2 + "  
 f"{beta\_i[3]:.5f} \* x3 + {beta\_i[4]:.5f} \* x1 \* x2 + "  
 f"{beta\_i[5]:.5f} \* x1 \* x3 + {beta\_i[6]:.5f} \* x2 \* x3 + {beta\_i[7]:.5f} \* x1 \* x2 \* x3")  
 check\_i = [round(sum(beta\_i[j] \* i[j] for j in range(len(beta\_i))), 3) for i in norm\_x]  
  
 if n == 4:  
 print(  
 f"\nThe normalized regression equation: "  
 f"y = {beta\_i[0]:.5f} + {beta\_i[1]:.5f} \* x1 + {beta\_i[2]:.5f} \* x2 + "  
 f"{beta\_i[3]:.5f} \* x3")  
 check\_i = [round(sum(beta\_i[j] \* i[j] for j in range(len(beta\_i))), 3) for i in norm\_x]  
  
 for i in range(len(check\_i)):  
 print(f'ŷ{i + 1} = {check\_i[i]}, y\_av{i + 1} = {y\_av[i]}')  
  
 print("\n[ Fisher's test ]")  
 f\_4 = n - d  
 s2\_ad = m / f\_4 \* sum([(check\_i[i] - y\_av[i]) \*\* 2 for i in range(len(y\_av))])  
 f\_p = s2\_ad / s2\_b  
 f\_t = {  
 1: [164.4, 199.5, 215.7, 224.6, 230.2, 234, 235.8, 237.6],  
 2: [18.5, 19.2, 19.2, 19.3, 19.3, 19.3, 19.4, 19.4],  
 3: [10.1, 9.6, 9.3, 9.1, 9, 8.9, 8.8, 8.8],  
 4: [7.7, 6.9, 6.6, 6.4, 6.3, 6.2, 6.1, 6.1],  
 5: [6.6, 5.8, 5.4, 5.2, 5.1, 5, 4.9, 4.9],  
 6: [6, 5.1, 4.8, 4.5, 4.4, 4.3, 4.2, 4.2],  
 7: [5.5, 4.7, 4.4, 4.1, 4, 3.9, 3.8, 3.8],  
 8: [5.3, 4.5, 4.1, 3.8, 3.7, 3.6, 3.5, 3.5],  
 9: [5.1, 4.3, 3.9, 3.6, 3.5, 3.4, 3.3, 3.3],  
 10: [5, 4.1, 3.7, 3.5, 3.3, 3.2, 3.1, 3.1],  
 11: [4.8, 4, 3.6, 3.4, 3.2, 3.1, 3, 3],  
 12: [4.8, 3.9, 3.5, 3.3, 3.1, 3, 2.9, 2.9],  
 13: [4.7, 3.8, 3.4, 3.2, 3, 2.9, 2.8, 2.8],  
 14: [4.6, 3.7, 3.3, 3.1, 3, 2.9, 2.8, 2.7],  
 15: [4.5, 3.7, 3.3, 3.1, 2.9, 2.8, 2.7, 2.7],  
 16: [4.5, 3.6, 3.2, 3, 2.9, 2.7, 2.6, 2.6],  
 17: [4.5, 3.6, 3.2, 3, 2.8, 2.7, 2.5, 2.3],  
 18: [4.4, 3.6, 3.2, 2.9, 2.8, 2.7, 2.5, 2.3],  
 19: [4.4, 3.5, 3.1, 2.9, 2.7, 2.7, 2.4, 2.3],  
 range(20, 22): [4.4, 3.5, 3.1, 2.8, 2.7, 2.7, 2.4, 2.3],  
 range(22, 24): [4.3, 3.4, 3.1, 2.8, 2.7, 2.6, 2.4, 2.3],  
 range(24, 26): [4.3, 3.4, 3, 2.8, 2.6, 2.5, 2.3, 2.2],  
 range(26, 28): [4.2, 3.4, 3, 2.7, 2.6, 2.5, 2.3, 2.2],  
 range(28, 30): [4.2, 3.3, 3, 2.7, 2.6, 2.4, 2.3, 2.1],  
 range(30, 40): [4.2, 3.3, 3, 2.7, 2.6, 2.4, 2.3, 2.1, 2, 2, 2, 2],  
 range(40, 60): [4.1, 3.2, 2.9, 2.6, 2.5, 2.3, 2.2, 2, 1.9, 1.9, 1.9, 1.9],  
 range(60, 120): [4, 3.2, 2.8, 2.5, 2.4, 2.3, 2.1, 1.9, 1.8, 1.8, 1.8, 1.8, 1.8, 1.8, 1.8, 1.8],  
 range(120, 2 \*\* 100): [3.8, 3, 2.6, 2.4, 2.2, 2.1, 2, 2, 1.9, 1.9, 1.9, 1.8, 1.8]  
 }  
 if f\_p > get\_value(f\_t, f\_3)[f\_4]:  
 n = 8 if n == 4 else 15  
 print(  
 f"fp = {f\_p} > ft = {get\_value(f\_t, f\_3)[f\_4]}.\n"  
 f"The mathematical model is not adequate to the experimental data\n"  
 f"Start again with m = m + 1 = {m + 1} and n = {n}")  
 return main(m=m + 1, n=n)  
 else:  
 print(  
 f"fP = {f\_p} < fT = {get\_value(f\_t, f\_3)[f\_4]}.\n"  
 f"The mathematical model is adequate to the experimental data\n")  
  
  
# n = 15 because if you start with 4 then it will not reach 15  
main(m=3, n=15)

Результати виконання роботи

ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3 + b12 \* x1 \* x2 + b13 \* x1 \* x3 + b23 \* x2 \* x3 + b123 \* x1 \* x2 \* x3 + b11 \* x1 \* x1 + b22 \* x2 \* x2 + b33 \* x3 \* x3



The naturalized regression equation: y = 198.42600 + -0.67200 \* x1 + 0.14600 \* x2 + -0.07600 \* x3 + -0.04700 \* x1 \* x2 + 0.05900 \* x1 \* x3 + -0.00000 \* x2 \* x3 + 0.01000 \* x1 \* x2 \* x3 + 0.28000 \* x1 \* x1 + 0.01600 \* x2 \* x2 + -0.03200 \* x3 \* x3

ŷ1 = 197.918, y\_av1 = 198.0

ŷ2 = 198.9, y\_av2 = 199.0

ŷ3 = 199.365, y\_av3 = 199.333

ŷ4 = 197.813, y\_av4 = 198.0

ŷ5 = 198.018, y\_av5 = 198.0

ŷ6 = 198.8, y\_av6 = 199.0

ŷ7 = 199.28, y\_av7 = 199.333

ŷ8 = 199.248, y\_av8 = 199.333

ŷ9 = 198.53, y\_av9 = 198.333

ŷ10 = 199.158, y\_av10 = 199.0

ŷ11 = 198.365, y\_av11 = 198.333

ŷ12 = 198.419, y\_av12 = 198.0

ŷ13 = 196.969, y\_av13 = 196.667

ŷ14 = 197.701, y\_av14 = 197.667

ŷ15 = 197.914, y\_av15 = 198.333

[ Kohren's test ]

The variance is homogeneous: Gp = 0.15528 < Gt = 0.6841

[ Student's test ]

The naturalized simplified regression equation: y = 198.42600 + -0.67200 \* x1 + 0.00000 \* x2 + 0.00000 \* x3 + 0.00000 \* x1 \* x2 + 0.00000 \* x1 \* x3 + 0.00000 \* x2 \* x3 + 0.00000 \* x1 \* x2 \* x3 + 0.00000 \* x1 \* x1 + 0.00000 \* x2 \* x2 + 0.00000 \* x3 \* x3

ŷ1 = 198.426, y\_av1 = 198.0

ŷ2 = 198.426, y\_av2 = 199.0

ŷ3 = 196.41, y\_av3 = 199.333

ŷ4 = 196.41, y\_av4 = 198.0

ŷ5 = 198.426, y\_av5 = 198.0

ŷ6 = 198.426, y\_av6 = 199.0

ŷ7 = 196.41, y\_av7 = 199.333

ŷ8 = 196.41, y\_av8 = 199.333

ŷ9 = 198.643, y\_av9 = 198.333

ŷ10 = 196.193, y\_av10 = 199.0

ŷ11 = 197.418, y\_av11 = 198.333

ŷ12 = 197.418, y\_av12 = 198.0

ŷ13 = 197.418, y\_av13 = 196.667

ŷ14 = 197.418, y\_av14 = 197.667

ŷ15 = 197.418, y\_av15 = 198.333

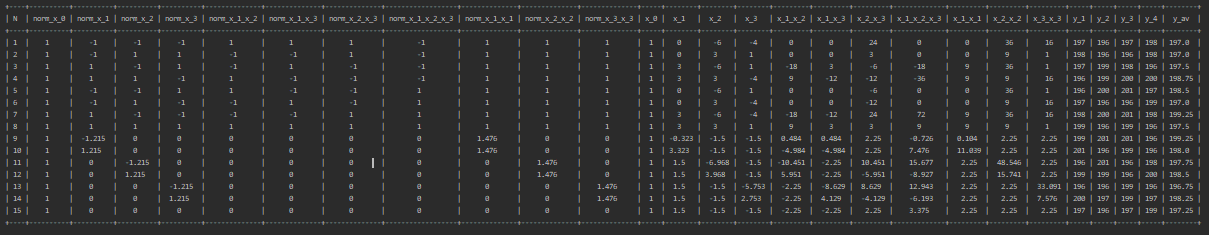
[ Fisher's test ]

fp = 5.561596818345413 > ft = 2.

The mathematical model is not adequate to the experimental data

Start again with m = m + 1 = 4 and n = 15

ŷ = b0 + b1 \* x1 + b2 \* x2 + b3 \* x3 + b12 \* x1 \* x2 + b13 \* x1 \* x3 + b23 \* x2 \* x3 + b123 \* x1 \* x2 \* x3 + b11 \* x1 \* x1 + b22 \* x2 \* x2 + b33 \* x3 \* x3



The naturalized regression equation: y = 198.13900 + -0.37200 \* x1 + -0.10800 \* x2 + 0.11900 \* x3 + 0.04100 \* x1 \* x2 + -0.12800 \* x1 \* x3 + -0.03300 \* x2 \* x3 + 0.01500 \* x1 \* x2 \* x3 + 0.11000 \* x1 \* x1 + -0.00300 \* x2 \* x2 + -0.02900 \* x3 \* x3

ŷ1 = 196.947, y\_av1 = 197.0

ŷ2 = 197.779, y\_av2 = 197.0

ŷ3 = 197.449, y\_av3 = 197.5

ŷ4 = 198.483, y\_av4 = 198.75

ŷ5 = 198.967, y\_av5 = 198.5

ŷ6 = 197.244, y\_av6 = 197.0

ŷ7 = 198.699, y\_av7 = 199.25

ŷ8 = 197.773, y\_av8 = 197.5

ŷ9 = 198.055, y\_av9 = 199.25

ŷ10 = 198.5, y\_av10 = 198.0

ŷ11 = 197.941, y\_av11 = 197.75

ŷ12 = 197.703, y\_av12 = 198.5

ŷ13 = 197.261, y\_av13 = 196.75

ŷ14 = 197.514, y\_av14 = 198.25

ŷ15 = 197.912, y\_av15 = 197.25

[ Kohren's test ]

The variance is homogeneous: Gp = 0.12926 < Gt = 0.6287

[ Student's test ]

The naturalized simplified regression equation: y = 198.13900 + 0.00000 \* x1 + 0.00000 \* x2 + 0.00000 \* x3 + 0.00000 \* x1 \* x2 + 0.00000 \* x1 \* x3 + 0.00000 \* x2 \* x3 + 0.00000 \* x1 \* x2 \* x3 + 0.00000 \* x1 \* x1 + 0.00000 \* x2 \* x2 + 0.00000 \* x3 \* x3

ŷ1 = 198.139, y\_av1 = 197.0

ŷ2 = 198.139, y\_av2 = 197.0

ŷ3 = 198.139, y\_av3 = 197.5

ŷ4 = 198.139, y\_av4 = 198.75

ŷ5 = 198.139, y\_av5 = 198.5

ŷ6 = 198.139, y\_av6 = 197.0

ŷ7 = 198.139, y\_av7 = 199.25

ŷ8 = 198.139, y\_av8 = 197.5

ŷ9 = 198.139, y\_av9 = 199.25

ŷ10 = 198.139, y\_av10 = 198.0

ŷ11 = 198.139, y\_av11 = 197.75

ŷ12 = 198.139, y\_av12 = 198.5

ŷ13 = 198.139, y\_av13 = 196.75

ŷ14 = 198.139, y\_av14 = 198.25

ŷ15 = 198.139, y\_av15 = 197.25

[ Fisher's test ]

fP = 1.8465462118492055 < fT = 1.9.

The mathematical model is adequate to the experimental data